ter place on the 25th. Floating ice was observed as far south as Boonville, Mo., on the 7th. The James River at Huron, S. Dak., closed on the 3d, while the Red River of the North, at Moorhead, Minn., was closed during the entire month.

The Mississippi River closed at Fort Ripley, Minn., on the 6th; at St. Paul, Minn., on the 2d; at Red Wing, Minn., on the 1st; at La Crosse, Wis., on the 7th, and at Prairie du Chien, Wis., on the 2d. At Dubuque, Iowa, the ice gorged above the bridge on the 14th, but remained open below. At Leclaire, Iowa, the river closed on the 7th.

The rivers of Maine were closed after the 4th or 5th, while the Connecticut closed at Wells River, Vt., on the 3d, and at Whiteriver Junction, Vt., on the 9th.

The highest and lowest water, mean stage, and monthly range at 207 river stations are given in Table IV. Hydrographs for typical points on seven principal rivers are shown on Chart I. The stations selected for charting are Keokuk, St. Louis, Memphis, Vicksburg, and New Orleans, on the Mississippi; Cincinnati and Cairo, on the Ohio; Nashville, on the Cumberland; Johnsonville, on the Tennessee; Kansas City, on the Missouri; Little Rock, on the Arkansas; and Shreveport, on the Red.—*H. C. Frankenfield, Professor of Meteorology.*

## SPECIAL ARTICLES, NOTES, AND EXTRACTS.

### STUDIES ON THE VORTICES OF THE ATMOSPHERE OF THE EARTH.

By Prof. FRANK H. BIGELOW. Dated Washington, D. C., March 16, 1908.

### V.—THE IMPERFECT TRUNCATED DUMB-BELL-SHAPED VORTEX AND THE COMPOSITION OF VORTICES ILLUSTRATED BY THE OCEAN-CYCLONE OF OCTOBER 11, 1905.

#### THE METEOROLOGICAL DATA.

A close examination of the isobars in the DeWitte typhoon[1] shows that there is a tendency to depart from the spacing which is required to produce the geometrical proportions called for by the formulas of the dumb-bell-shaped vortex. This feature is perceived first on the outer and on the inner isobars rather than in the middle ones of the group, and the general result is to change from a geometrical ratio for the radii to such a spacing as gives an equal distance between the successive radii of the series. This is equivalent to saying that the hyperbolic law, $v\varpi$=constant, which has been employed in the dumb-bell-shaped vortex, tends to become the parabolic law, $\frac{v}{\varpi}$ = constant. (Compare the Cloud Report, pages 509, 620.) In the hyperbolic type of motion the gyration of the inner tubes is much greater than that of the outer tubes; in the parabolic type of motion the inner tangential velocities are not so great relatively as in the hyperbolic type. It is the purpose of this, and the following paper on the land-cyclone, to study the facts regarding this transition, and the thermodynamic causes which produce them. In the ocean-cyclone with strong development we find evidences of both of these types of motion in combination, and in the land-cyclone the parabolic type seems to be in the lead thruout the area of the local circulation. The subject is much complicated by the fact that the cyclone is generated by independent streams of warm and cold air which underrun the powerful eastward drift. This feature is influential in modifying the types of motion from pure forms to others that are quite incapable of being reduced to mathematical analysis of any simple kind.

The synoptic weather chart of the ocean-cyclone of October 11, 1905,[2] constructed by Mr. James Page (see fig. 17), contains much of the data required for study. Table 65 contains the geographical positions of 110 ships which rendered reports for October 11, 1905, at Greenwich mean noon, the wind direction and velocity, the barometric pressure and the temperature, all reduced to metric measures from extracts of the original records prepared by myself for this paper.

In order to eliminate the local conditions and secure the data for a symmetrical vortex about a central axis, we proceeded as follows: The linear diameters of the isobars were scaled on fig. 17, first in the northwest-southeast direction and then in the southwest-northeast direction, and the mean radii computed. Table 66 gives the computations from step to step.

[1] See Monthly Weather Review, October, 1908, **36**, p. 328, and Chart IX.

[2] See Monthly Weather Review, January, 1906, **34**, p. 5, fig. 3.

The columns contain in succession: (1) the barometric pressure in inches; (2), (3), the diameters of the isobars in two directions about 90° apart; (4) the sum of (2) and (3); (5) the

TABLE 65.—*Temperatures observed in the ocean-cyclone of October 11, 1905, at Greenwich mean noon.*

| No. | Latitude N. | Longitude W. | Wind. Direction. | Wind. Meters per second. | *B* | *T* |
|---|---|---|---|---|---|---|
| | | | | | *Mm.* | °*C.* |
| 1.. | ...... | ...... | e. | 8 | 762 | 10.0 |
| 2.. | ...... | ...... | ...... | ... | ..... | ..... |
| 3.. | 45.5 | 55.3 | nne. | 22 | 747 | 5.6 |
| 4.. | 50.6 | 27.3 | sse. | 13 | 766 | 14.6 |
| 5.. | 19.9 | 56.9 | w. | 6 | 758 | 28.0 |
| 6.. | 47.1 | 46.8 | ne. | 18 | 744 | 9.4 |
| 7.. | 41.6 | 42.6 | se. & e. tosw. | 40 | 719 | 20.6 |
| 8. | 35.6 | 51.5 | n. | 22 | 744 | 20.0 |
| 9.. | 41.9 | 64.8 | nne. | 10 | 767 | 9.5 |
| 10.. | 39.6 | 49.6 | nnw. | 18 | 753 | 13.9 |
| 11.. | 42.8 | 61.0 | ne. | 13 | 768 | 10.0 |
| 12.. | 41.5 | 65.5 | nne. | 6 | 758 | 14.4 |
| 13.. | 49.8 | 26.1 | sse. | 10 | 766 | 15.5 |
| 14.. | 50.0 | 26.8 | s. | 10 | 767 | 13.9 |
| 15.. | 39.0 | 55.7 | n. | 18 | 756 | 12.8 |
| 16.. | 44.9 | 49.6 | ne. | 25 | 761 | 9.5 |
| 17.. | 50.7 | 25.6 | se. | 10 | 767 | 15.0 |
| 18.. | 50.8 | 43.4 | ne. | 15 | 748 | 13.3 |
| 19.. | 47.0 | 46.1 | n. | 22 | 746 | 10.0 |
| 20.. | 41.0 | 42.8 | wsw. | 34 | 720 | 17.8 |
| 21.. | 40.8 | 28.9 | se. | 13 | 761 | 21.1 |
| 22.. | 51.6 | 81.5 | sse. | 13 | 755 | 18.9 |
| 23.. | 48.4 | 49.1 | nne. | 18 | 747 | 7.3 |
| 25.. | 28.5 | 68.2 | e. & n. | 6 | 770 | 24.4 |
| 26.. | 45.1 | 48.9 | ene. | 29 | 743 | 8.4 |
| 27.. | 42.7 | 58.0 | n. | 13 | 761 | 10.0 |
| 28.. | 81.9 | 58.8 | n. | 4 | 758 | 24.0 |
| 29.. | 42.7 | 61.7 | ne. | 13 | 764 | 10.0 |
| 30.. | 45.8 | 49.9 | nne. | 25 | 744 | 9.0 |
| 31.. | 49.1 | 37.4 | s. | 5 | 752 | 18.4 |
| 32.. | 28.0 | 79.5 | ene. | 13 | 762 | 29.7 |
| 33.. | ...... | ...... | ...... | ... | ..... | ..... |
| 34.. | 20.2 | 61.7 | nw. | 6 | 764 | 24.6 |
| 35.. | 43.9 | 54.8 | nne. | 22 | 757 | 7.8 |
| 36.. | ...... | ...... | ...... | ... | ..... | ..... |
| 37 . | 42.9 | 58.6 | ne. | 15 | 755 | 7.0 |
| 38.. | 50.4 | 30.3 | sse. | 10 | 764 | 15.6 |
| 39.. | 42.3 | 44.0 | n. | 40 | 719 | 18.0 |
| 40 . | ...... | ...... | ...... | ... | ..... | ..... |
| 41.. | 21.7 | 67.0 | nne. | 8 | 763 | 27.7 |
| 42.. | 26.0 | 66.9 | ene. | 8 | 760 | 26.0 |
| 43.. | 49.5 | 27.4 | sse. | 10 | 765 | 14.5 |
| 44.. | 36.6 | 59.0 | n. | 23 | 763 | 19.5 |
| 45.. | Horta, Azores. | | se. | 9 | 763 | 21.5 |
| 46.. | 50.4 | 29.5 | sse. | 10 | 765 | 14.4 |
| 47.. | 40 0 | 19.0 | se. | 10 | 762 | 13.9 |
| 48.. | 41.7 | 45.4 | nnw. | 40 | 716 | 16.7 |
| 49.. | 47.5 | 26.7 | se. | 4 | 764 | 16.2 |
| 50.. | 45.5 | 84.3 | sse. | 22 | 751 | 21.1 |
| 51.. | 41.0 | 66.3 | nne. | 8 | 766 | 13.0 |
| 52.. | 35.8 | 40.3 | s. 60 w. | 22 | 749 | 24.4 |
| 53.. | 49.1 | 84.1 | sse. | 18 | 759 | 16.2 |
| 54.. | 40.1 | 68.1 | ne. | 8 | 767 | 12.5 |
| 55.. | 49.8 | 12.0 | ene. | 10 | 773 | 15.0 |
| 56.. | 45.3 | 43.2 | ene. | 29 | 732 | 16.0 |
| 57.. | 49.5 | 82.0 | se. by s. | 18 | 753 | 16.2 |
| 58.. | 49.5 | 42.2 | ne. | 6 | 750 | 12.8 |
| 59.. | Halifax | ...... | nw. | 3 | 765 | 6.7 |
| 60.. | 40.3 | 30.0 | s. | 18 | 736 | 20.6 |
| 58a | 36.1 | 42.1 | wsw. | 22 | 732 | 25.5 |

| No. | Latitude N. | Longitude W. | Wind. Direction. | Wind. Meters per second. | *B* | *T* |
|---|---|---|---|---|---|---|
| | | | | | *Mm.* | °*C.* |
| 61. | 40.3 | 47.3 | wnw. | 22 | 740 | 11.1 |
| 62. | 30.0 | 67.4 | ene. | 4 | 760 | 24.4 |
| 63. | 43.9 | 56.2 | n. | 22 | 762 | 8.9 |
| 64. | 42.4 | 52.5 | nnw. | 29 | 755 | 11.1 |
| 65. | 49.8 | 30.0 | s. | 15 | 760 | 14.4 |
| 66. | 42.3 | 59.0 | nne. | 22 | 757 | 10.0 |
| 67. | 40.5 | 83.5 | s. | 18 | 761 | 22.2 |
| 68. | 45.2 | 47.0 | ne. | 25 | 727 | 8.9 |
| 69. | 43.7 | 41.9 | sse. | 2 | 719 | 14.0 |
| 70. | 22.4 | 84.5 | sw. | 13 | 764 | 25.0 |
| 71. | 49.8 | 36.4 | s. & e. | 18 | 778 | 15.0 |
| 72. | 44.6 | 49.8 | n. | 87 | 743 | 8.9 |
| 73. | 50.2 | 28.3 | s. | 10 | 766 | 15.8 |
| 74. | 36.2 | 66.8 | wnw. | 10 | 758 | 20.0 |
| 75. | 40.6 | 73.1 | ese. | 13 | 761 | 17.5 |
| 76. | 48.0 | 38.1 | se. | 25 | 747 | 17.3 |
| 77. | ...... | ...... | ...... | ... | ..... | ..... |
| 78. | 40.2 | 23.9 | ene. | 4 | 761 | 20.6 |
| 79. | 49.2 | 44.5 | ne. | 15 | 740 | 10.0 |
| 80. | 25.7 | 68.2 | e. & s. | 6 | 761 | 27.2 |
| 81. | 42.6 | 60.0 | ne. | 15 | 762 | 10.0 |
| 82. | 17.3 | 72.4 | e. | 6 | 758 | — 1.0 |
| 83. | 25.1 | 38.9 | sse. | 13 | 767 | — 2.2 |
| 84. | 43.8 | 53.7 | n. | 25 | 755 | 11.1 |
| 85. | 54.0 | 22.5 | se. | 10 | 770 | 14.2 |
| 86 | 48.4 | 40.9 | ne. | 16 | 741 | 15.0 |
| 87. | 42.5 | 48.5 | n. | 29 | 738 | 8.3 |
| 88. | Santa Cruz, Az. | | se. | 2 | 760 | 20.7 |
| 89. | ...... | ...... | ...... | ... | ..... | ..... |
| 90. | 48.2 | 51.3 | n. | 18 | 751 | 6.7 |
| 91. | ...... | ...... | ...... | ... | ..... | ..... |
| 92. | 44.3 | 53.9 | nne. | 18 | 758 | —10.6 |
| 93. | 51.2 | 19.0 | sse. | 10 | 766 | 15.6 |
| 94. | 41.8 | 60.4 | ne. | 15 | 761 | 9.4 |
| 95. | 16.0 | 47.3 | se. & s. | 10 | 760 | 26.6 |
| 96. | 27.1 | 73.7 | Var. | 8 | 762 | 26.0 |
| 97. | 30.8 | 52.4 | nnw. | 15 | 752 | 23.3 |
| 98. | 40.2 | 48.0 | n. | 29 | 738 | 12.8 |
| 99. | 51.6 | 44.3 | nne. | 13 | 745 | 10.6 |
| 100. | ...... | ...... | ...... | ... | ..... | ..... |
| 101. | 38.5 | 22.0 | ne. | 10 | 763 | 18.9 |
| 102. | 44.7 | 46.7 | ne. | 40 | 703 | 13.3 |
| 103. | 35.3 | 45.8 | w. | 18 | 734 | 20.0 |
| 104. | 24.2 | 68.4 | ene. | 8 | 763 | — 3.3 |
| 105. | 43.8 | 56.2 | n. | 18 | 756 | 6.7 |
| 106. | 46.5 | 47.0 | nne. | 22 | 744 | 9.4 |
| 107. | 49.5 | 81.0 | s. | 15 | 757 | 13.9 |
| 108. | 49.9 | 41.5 | n. | 2 | 751 | —11.1 |
| 109. | 38.8 | 69.6 | e. | 8 | 763 | 18.9 |
| 110. | 49.5 | 82.8 | s. | 13 | 755 | 16.2 |

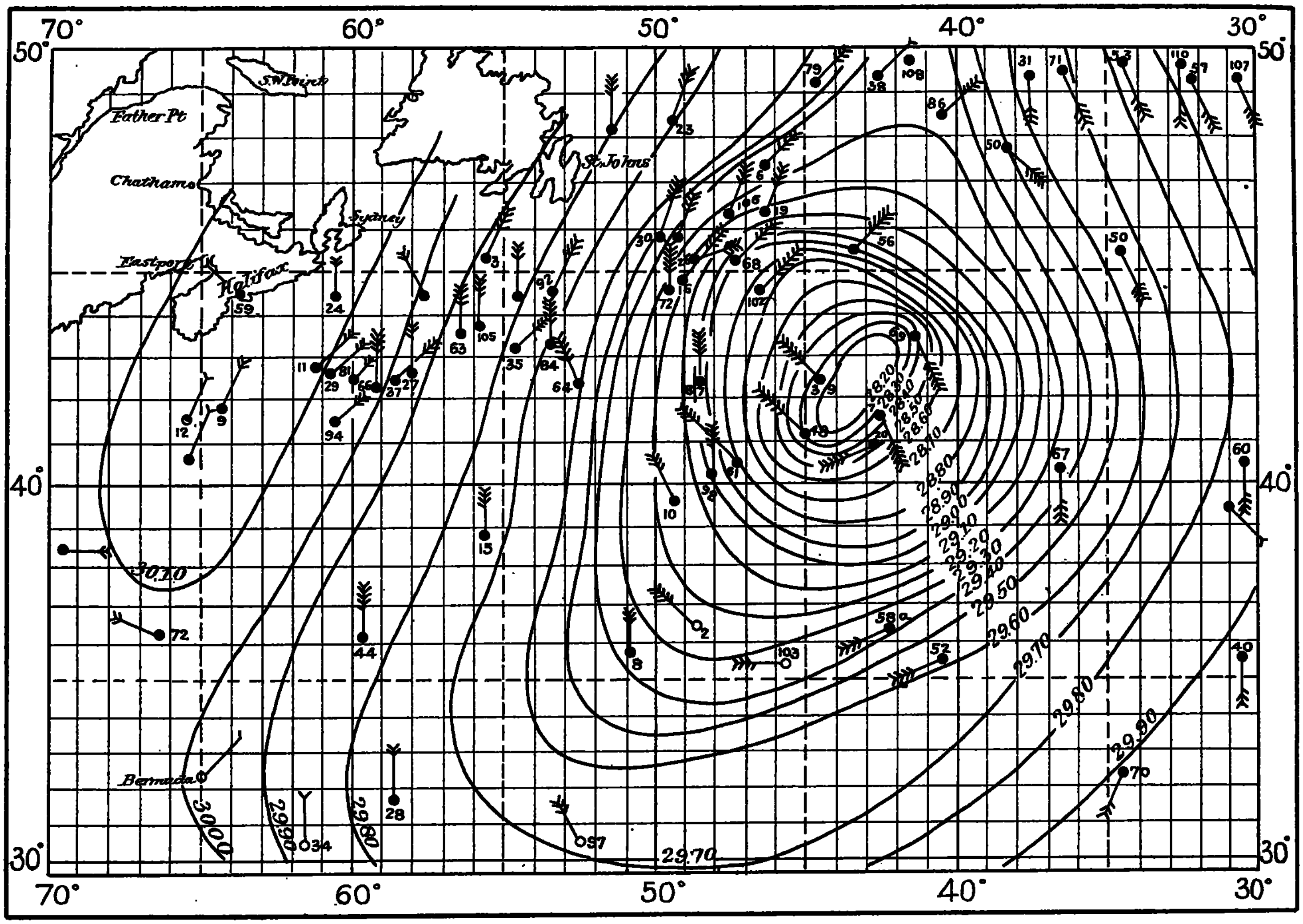


FIG. 17.—Synoptic weather chart of the North Atlantic, Greenwich mean noon, October 11, 1905. (Mercator Projection.)

mean radius $\varpi$; and (6) the difference $\Delta\varpi$. These values were plotted on a diagram and after drawing a mean curve, the smoothed results were placed in (7) and (8). The log $\varpi$ and log $\rho$ are given in (9) and (10). The metric equivalent of $B$ in inches is given in (11); the values of $B$ for the intervals beginning with 760 millimeters in (12) as interpolated from $B$ in (11); and the corresponding smoothed $\varpi$ in (13).

The last two columns of Table 66, giving $B$ and $\varpi$, are transferred to Table 68, where the adjustment of the radii and the velocities are computed. From log $\varpi_n$ is found $\log \rho = \log \varpi_n - \log \varpi_{n+1}$. The values of log $\rho$ are nearly constant from $B = 755.0$ to $B = 730.0$, while they are somewhat larger above 755.0 and gradually increase below 730.0. This shows that the radii, as scaled from the chart, do not conform to a logarithmic law, except in the middle group of isobars. This may be due partly to imperfect data, because the drawing of the isobars, especially near the center, may be inaccurate, and partly to the fact that the vortex is really impure and departs from the true vortex law in consequence of the system of forces that has generated it, especially the asymmetric distribution of the temperature which begins to show at the surface in the ocean-cyclone, and to distinguish these vortices from the more perfect vortices observed in hurricanes. We find the mean,

$$\log \rho_m = 0.05354 \text{ for 2.5 mm. intervals.}$$

The velocities were next considered, and a collection of the observed tangential velocities $v$, on or near each isobar, was made, so that the mean tangential velocity on each isobar could be computed. See Table 67.

TABLE 66.—*Adjustment of the isobars and the radii to a symmetrical type about the z-axis.*

| $B$ | Diameters of isobars. NW. | Diameters of isobars. SW. | Diameters of isobars. Sum | Diameters of isobars. $\varpi$ | $\Delta\varpi$ | Adjusted. $\Delta\varpi$ | Adjusted. $\varpi$ | log $\varpi$ | log $\rho$ | $B$ | Adjusted. $B$ | Adjusted. $\varpi$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Ins.* | *Mm.* | *Mm.* | *Mm.* | *Mm.* | *Mm.* | *Mm.* | *Mm.* | | | *Mm.* | *Mm.* | *Mm.* |
| 29.90 | 126 | 175 | 301 | 75.3 | | | 75.3 | 1.87679 | | 759.5 | 760.0 | 76.0 |
| | | | | | 9.8 | 10.8 | | | 0.06723 | | | |
| 29.80 | 106 | 156 | 262 | 65.5 | | | 64.5 | 1.80956 | | 756.9 | 757.5 | 66.4 |
| | | | | | 9.9 | 8.5 | | | 0.06137 | | | |
| 29.70 | 93 | 130 | 223 | 55.6 | | | 56.9 | 1.74819 | | 754.4 | 755.0 | 57.0 |
| | | | | | 8.1 | 6.9 | | | 0.05711 | | | |
| 29.60 | 82 | 108 | 190 | 47.5 | | | 49.1 | 1.69108 | | 751.8 | 752.5 | 50.4 |
| | | | | | 4.5 | 5.8 | | | 0.05459 | | | |
| 29.50 | 76 | 96 | 172 | 43.0 | | | 43.3 | 1.63649 | | 749.3 | 750.0 | 45.0 |
| | | | | | 4.5 | 5.2 | | | 0.05557 | | | |
| 29.40 | 69 | 85 | 154 | 38.5 | | | 38.1 | 1.58092 | | 746.8 | 747.5 | 39.8 |
| | | | | | 4.0 | 4.5 | | | 0.05458 | | | |
| 29.30 | 63 | 75 | 138 | 34.5 | | | 33.6 | 1.52634 | | 744.2 | 745.0 | 35.0 |
| | | | | | 4.0 | 3.8 | | | 0.05212 | | | |
| 29.20 | 58 | 64 | 122 | 30.5 | | | 29.8 | 1.47422 | | 741.7 | 742.5 | 31.2 |
| | | | | | 4.0 | 3.4 | | | 0.05262 | | | |
| 29.10 | 54 | 52 | 106 | 26.5 | | | 26.4 | 1.42160 | | 739.1 | 740.0 | 27.8 |
| | | | | | 2.5 | 3.0 | | | 0.05168 | | | |
| 29.00 | 50 | 46 | 96 | 24.0 | | | 23.4 | 1.36922 | | 736.6 | 737.5 | 24.5 |
| | | | | | 1.7 | 2.7 | | | 0.05325 | | | |
| 28.90 | 46 | 43 | 89 | 22.3 | | | 20.7 | 1.31597 | | 734.1 | 735.0 | 21.6 |
| | | | | | 3.0 | 2.4 | | | 0.05352 | | | |
| 28.80 | 39 | 38 | 77 | 19.3 | | | 18.3 | 1.26245 | | 731.5 | 732.5 | 19.4 |
| | | | | | 2.8 | 2.3 | | | 0.05833 | | | |
| 28.70 | 33 | 33 | 66 | 16.5 | | | 16.0 | 1.20412 | | 729.0 | 730.0 | 17.0 |
| | | | | | 2.2 | 2.1 | | | 0.06111 | | | |
| 28.60 | 26 | 31 | 57 | 14.3 | | | 13.9 | 1.14301 | | 726.4 | 727.5 | 14.8 |
| | | | | | 1.8 | 2.0 | | | 0.07113 | | | |
| 28.50 | 21 | 29 | 50 | 12.5 | | | 11.8 | 1.07188 | | 723.9 | 725.0 | 12.7 |
| | | | | | 2.5 | 2.0 | | | 0.08065 | | | |
| 28.40 | 16 | 24 | 40 | 10.0 | | | 9.8 | 0.99123 | | 721.4 | 722.5 | 10.6 |
| | | | | | 2.0 | 2.0 | | | 0.09914 | | | |
| 28.30 | 11 | 21 | 32 | 8.0 | | | 7.8 | 0.89209 | | 718.8 | 720.0 | 8.7 |
| | | | | | 2.2 | 2.0 | | | 0.12866 | | | |
| 28.20 | 7 | 16 | 23 | 5.8 | | | 5.8 | 0.76843 | | 716.8 | 717.5 | 6.6 |

TABLE 67.—*The observed tangential velocities, v, in meters per second.*

| Isobars. | 760 | 755 | 750 | 745 | 740 | 735 | 730 | 725 | 720 |
|---|---|---|---|---|---|---|---|---|---|
| Velocities observed at points along the isobars. | 13 | 15 | 22 | 22 | 22 | 25 | 40 | 34 | 40 |
| | 10 | 18 | 25 | 34 | 22 | 22 | 25 | | 37 |
| | 15 | 18 | 29 | 22 | 25 | 25 | | | 40 |
| | 15 | 22 | 22 | 25 | 22 | | | | 40 |
| | 22 | 18 | 22 | 15 | 34 | | | | |
| | 18 | 22 | 22 | 18 | 18 | | | | |
| | 10 | 29 | | 22 | | | | | |
| | 25 | 18 | | 18 | | | | | |
| | 8 | 10 | | 18 | | | | | |
| | 13 | 15 | | | | | | | |
| | 13 | 6 | | | | | | | |
| | 5 | | | | | | | | |
| | 18 | | | | | | | | |
| Means | 14.2 | 17.4 | 23.7 | 21.6 | 23.8 | 24.0 | 32.5 | 34.0 | 39.3 |
| Smoothed | 14.2 | 16.5 | 19.0 | 21.5 | 24.5 | 27.6 | 31.0 | 33.0 | 38.0 |

The mean values of $v$ were plotted on a diagram, and the average curve drawn thru the points gave the smoothed values of $v$ in the lowest line. This is transferred to column (6), Table 68. The log $v$ is taken, and $\log \sigma = \log v_{n+1} - \log v_n$ computed for 5.0 mm. intervals.

TABLE 68.—*Determination of log $\rho$ from the smoothed radii $\varpi_n$; and of log $\sigma$ from the smoothed velocities $v_n$.*

| $B$ | $\varpi$ | $\log \varpi$ | $\log \rho$ | | $v$ | $\log v$ | $\log \sigma$ | |
|---|---|---|---|---|---|---|---|---|
| *Mm.* | *Mm.* | | | Mean $\log \rho_m = 0.05354$ for 2.5 *mm.* intervals. Adopted $\log \rho_m = 0.05300$. | *M. p. s.* | | | Mean $\log \sigma = 0.05222$ for 5.0 *mm.* intervals. Adopted $\log \sigma = 0.05300$. |
| 760.0 | 76.0 | 1.88031 | | | 14.2 | 1.15229 | | |
| | | | 0.05864 | | | | | |
| 757.5 | 66.4 | 1.82217 | | | | | .06519 | |
| | | | 0.06630 | | | | | |
| 755.0 | 57.0 | 1.75587 | | | 16.5 | 1.21748 | | |
| | | | 0.05344 | | | | | |
| 752.5 | 50.4 | 1.70243 | | | | | .06127 | |
| | | | 0.04922 | | | | | |
| 750.0 | 45.0 | 1.65321 | | | 19.0 | 1.27875 | | |
| | | | 0.05333 | | | | | |
| 747.5 | 39.8 | 1.59988 | | | | | .05369 | |
| | | | 0.05581 | | | | | |
| 745.0 | 35.0 | 1.54407 | | | 21.5 | 1.33244 | | |
| | | | 0.04992 | | | | | |
| 742.5 | 31.2 | 1.49415 | | | | | .05573 | |
| | | | 0.05011 | | | | | |
| 740.0 | 27.8 | 1.44404 | | | 24.5 | 1.38917 | | |
| | | | 0.05487 | | | | | |
| 737.5 | 24.5 | 1.38917 | | | | | .05174 | |
| | | | 0.05472 | | | | | |
| 735.0 | 21.6 | 1.33445 | | | 27.6 | 1.44091 | | |
| | | | 0.05665 | | | | | |
| 732.5 | 19.4 | 1.28780 | | | | | .05045 | |
| | | | 0.05735 | | | | | |
| 730.0 | 17.0 | 1.23045 | | | 31.0 | 1.49136 | | |
| | | | 0.06019 | | | | | |
| 727.5 | 14.8 | 1.17026 | | | | | .04646 | |
| | | | 0.06646 | | | | | |
| 725.0 | 12.7 | 1.10880 | | | 34.5 | 1.53782 | | |
| | | | 0.07849 | | | | | |
| 722.5 | 10.6 | 1.02531 | | | | | .04196 | |
| | | | 0.08579 | | | | | |
| 720.0 | 8.7 | 0.93952 | | | 38.0 | 1.57978 | | |
| | | | 0.11998 | | | | | |
| 717.5 | 6.6 | 0.81954 | | | | | .04347 | |
| | | | 0.15678 | | | | | |
| 715.0 | 4.6 | 0.66276 | | | 42.0 | 1.62325 | | |

It is seen that while log $\sigma$ decreases, the range is not very great, and the mean value of log $\sigma$ for 5.0 mm. intervals is

$$\log \sigma_m = 0.05222.$$

The log $\sigma$ is taken for isobars at twice as great intervals as was used for computing log $\rho_m$, so that we may adopt the following average values,

$$\log \rho_m = 2 \log \sigma_m = 0.10600$$

when the isobars are 760, 755, 750, ..., 715. Adopt the values of $\varpi$ and $v$ at the 760 mm. isobar as the standard, and compute new values, using the average value of log $\rho_m$ thruout the system.

With the adopted constant logarithms of $\rho$ and $\sigma$ derived from Table 68, and the following formulas:

$$\log \varpi_{n+1} = \log \varpi_n - \log \rho = \log \varpi_n - 0.10600$$
$$\log v_{n+1} = \log v_n + \tfrac{1}{2} \log \rho = \log v_n + 0.05300$$
$$\log a\psi_{n+1} = \log a\psi_n - \tfrac{1}{2} \log \rho = \log a\psi_n - 0.05300$$

compute the values of log $\varpi$, log $v$, log $a\psi_n$ as given in columns 3, 7, and 8, of Table 69. The second column of the table gives the corresponding or final adjusted values of $\varpi$ appropriate to the original chart, fig. 17, whose scale is such that 1 mm. = 20000 m. = 20 km. = 12.4 miles.

The fourth column gives the logarithm of the equivalent $\varpi$ in meters on the surface of the globe, and the fifth column its value in kilometers.. The sixth column gives the adjusted value of $v$ in meters per second corresponding to the *adjusted* values of log $v$ in the seventh column. The computed values agree closely with those obtained from the chart and data of Table 65, except near the center.

TABLE 69.—*Adjusted values of log $\varpi_n$, log $v_n$, log $a\psi_n$.*

| $B$ | $\varpi$ | $\log \varpi$ | $\log \varpi$ | $\varpi$ | $v$ | $\log v$ | $\log a\psi_n$ |
|---|---|---|---|---|---|---|---|
| *Mm.* | *Mm.* | | | *Km.* | *M. p. s.* | | |
| 760 | 73.3 | 1.86522 | 6.16625 | 1466.4 | 14.9 | 1.17275 | 7.33900 |
| 755 | 57.4 | 1.75922 | 6.06025 | 1148.8 | 16.8 | 1.22575 | 7.28600 |
| 750 | 45.0 | 1.65322 | 5.95425 | 900.0 | 19.0 | 1.27875 | 7.23300 |
| 745 | 35.8 | 1.54722 | 5.84825 | 705.1 | 21.5 | 1.33175 | 7.18000 |
| 740 | 27.7 | 1.44122 | 5.74225 | 552.4 | 24.3 | 1.38475 | 7.12700 |
| 735 | 21.6 | 1.33522 | 5.63625 | 432.8 | 27.4 | 1.43775 | 7.07400 |
| 730 | 17.0 | 1.22922 | 5.53025 | 339.0 | 31.0 | 1.49075 | 7.02100 |
| 725 | 13.3 | 1.12322 | 5.42425 | 265.6 | 35.0 | 1.54375 | 6.96800 |
| 720 | 10.4 | 1.01722 | 5.31825 | 208.1 | 39.5 | 1.59675 | 6.91500 |
| 715 | 8.2 | 0.91122 | 5.21225 | 163.0 | 44.6 | 1.64975 | 6.86200 |

Our purpose is now to determine the average vortex system that is nearly equivalent to the observed ocean-cyclone, and compare it with the pure vortex from which it may be assumed to have departed. The differences between the perfect and the imperfect vortices will enable us to construct the *disturbing vortex* system that transforms the vortex of hurricane type into the observed cyclone. The thermodynamic forces which will produce such a disturbing vortex may be attributed to the asymmetric distribution of the air masses of different temperatures around the axis of gyration. It is noted in the last column of Table 69 that the $\log a\psi = \log \varpi v$ is not a constant as it should be in the perfect vortex, because the tangential velocities are not great enough along the inner isobars to conform to the vortex which is represented by the outer isobars. It will be necessary then to continue this computation with a *variable current function.*

COMPUTATIONS FOR $A$, $\varpi$, $u$, $v$, $w$, IN THE IMPERFECT AND IN THE PERFECT VORTICES ON THE PLANE OF REFERENCE $az = 50°$.

It was decided to begin the computations on the plane $az = 50°$, near the sea-level, to give an inflowing angle $i = -40°$. The angle probably lies between $-40°$ and $-30°$, and this will place the plane of greatest angular velocity in the strato-cumulus level, about 3,000 meters above the sea-level. At the same time the elevation of the upper plane is taken 8,000 meters above the sea-level, instead of 12,000 meters as in the DeWitte typhoon. This gives us,

$$a = \frac{180°}{8000 + 4000} = 0.015°,$$

as the angle-constant. It may be that this plane should be taken somewhat higher, but it is probable that the eastward drift into which this cyclone penetrates practically destroys the vortex head near that level. This is, of course, a point for a more careful research. Table 70 contains the computations of $A_1$, $\varpi_1$, $u_1$, $v_1$, $w_1$, in the simplest order for the imperfect vortex with a *variable* value of the current function $a\psi$. Table 71 contains a similar computation of $A_0$, $\varpi_0$, $u_0$, $v_0$, $w_0$, for the perfect vortex with a constant value of the current function. A comparison of the values of the velocities in Tables 70 and 71 shows how great a disturbance of the perfect vortex has been effected. It is the problem, in a correct theory of the cyclone, to account for these differences of the velocities. In Tables 72 and 73 the results are extended to the 10-degree values of $az$ from 50° to 180°, and these enable us to proceed with the discussion of the temperature distribution that is properly responsible for these motions of the atmosphere.

TABLE 70.—*The imperfect dumb-bell-shaped vortex.*

**Computations on the plane $az = 50°$.**

Current F. $\psi = A\varpi^2 \sin az$.

Velocities.

Radial. $u = -Aa\varpi \cos az$.

Tangential. $v = +Aa\varpi \sin az$.

Vertical. $w = +2A \sin az$.

Constants. $\log \rho = 0.10600$.
$\log \sigma = 0.05300$.

Radius $\varpi = \left( \dfrac{a\psi}{Aa \sin az} \right)^{\frac{1}{2}}$

Constant $A = \dfrac{v}{a\varpi \sin az}$

$\log \sin 50° = 9.88425$
$\log \cos 50° = 9.80807$

$a = \dfrac{180°}{8000 + 4000}$.

$a = \dfrac{180°}{12000} = 0.015°$.

$\log a = 8.17609$.

$\log a \sin az = 8.06034$.
$\log a \cos az = 7.93416$.

Variable $a\psi$. The values of $A_1$, $\varpi_1$, $u_1$, $v_1$, $w_1$.

| Term. | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
|---|---|---|---|---|---|---|---|---|---|---|
| $\log a\psi$ | 7.33900 | 7.28600 | 7.23300 | 7.18000 | 7.12700 | 7.07400 | 7.02100 | 6.96800 | 6.91500 | 6.86200 |
| $\log \varpi$ | 6.16625 | 6.06025 | 5.95425 | 5.84825 | 5.74225 | 5.63625 | 5.53025 | 5.42425 | 5.31825 | 5.21225 |
| $\varpi$ (meters) | 1466400. | 1148800. | 900020. | 705100. | 552400. | 432760. | 339040. | 265610. | 208090. | 163020. |
| $\log v_1$ | 1.17275 | 1.22575 | 1.27875 | 1.33175 | 1.38475 | 1.43775 | 1.49075 | 1.54375 | 1.59675 | 1.64975 |
| $v_1$ | 14.9 | 16.8 | 19.0 | 21.5 | 24.3 | 27.4 | 31.0 | 35.0 | 39.5 | 44.6 |
| $a\varpi \sin az$ | 4.22659 | 4.12059 | 4.01459 | 3.90859 | 3.80259 | 3.69659 | 3.59059 | 3.48459 | 3.37859 | 3.27259 |
| $\log A_1$ | 6.94616 | 7.10516 | 7.26416 | 7.42316 | 7.58216 | 7.74116 | 7.90016 | 8.05916 | 8.21816 | 8.37716 |
| $A_1$ | .00089 | .00127 | .00184 | .00265 | .00382 | .00551 | .00795 | .01146 | .01653 | .02383 |
| $\log u_1$ | —1.09657 | —1.14957 | —1.20257 | —1.25557 | —1.30857 | —1.36157 | —1.41457 | —1.46757 | —1.52057 | —1.57357 |
| $u_1$ | —12.5 | —14.1 | —16.0 | —18.0 | —20.4 | —23.0 | —26.0 | —29.4 | —33.2 | —37.5 |
| $\log w_1$ | 7.13144 | 7.29044 | 7.44944 | 7.60844 | 7.76744 | 7.92644 | 8.08544 | 8.24444 | 8.40344 | 8.56244 |
| $w_1$ | .00135 | .00195 | .00282 | .00406 | .00585 | .00844 | .01217 | .01756 | .02532 | .03651 |

TABLE 71.—*The perfect dumb-bell-shaped vortex.*

Constant $a\psi$. The values of $A_0$, $\varpi_0$, $u_0$, $v_0$, $w_0$.

| Term. | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
|---|---|---|---|---|---|---|---|---|---|---|
| $\log a\psi$ | 7.33900 | 7.33900 | 7.33900 | 7.33900 | 7.33900 | 7.33900 | 7.33900 | 7.33900 | 7.33900 | 7.33900 |
| $\log \varpi$ | 6.16625 | 6.06025 | 5.95425 | 5.84825 | 5.74225 | 5.63625 | 5.53025 | 5.42425 | 5.31825 | 5.21225 |
| $\varpi$ (meters) | 1466400. | 1148800. | 900020. | 705100. | 552400. | 432760. | 339040. | 265610. | 208090. | 163020. |
| $\log v_0$ | 1.17275 | 1.27875 | 1.38475 | 1.49075 | 1.59675 | 1.70275 | 1.80875 | 1.91475 | 2.02075 | 2.12675 |
| $v_0$ | 14.9 | 19.0 | 24.2 | 29.6 | 39.5 | 50.8 | 64.4 | 82.2 | 104.9 | 133.9 |
| $a\varpi \sin az$ | 4.22659 | 4.12059 | 4.01459 | 3.90859 | 3.80259 | 3.69659 | 3.59059 | 3.48459 | 3.37859 | 3.27259 |
| $\log A_0$ | 6.94616 | 7.15816 | 7.37016 | 7.58216 | 7.79416 | 8.00616 | 8.21816 | 8.43016 | 8.64216 | 8.85416 |
| $A_0$ | .00089 | .00144 | .00235 | .00382 | .00623 | .01014 | .01653 | .02699 | .04387 | .07148 |
| $\log u_0$ | —1.09657 | —1.20257 | —1.30857 | —1.41457 | —1.52057 | —1.62657 | —1.73257 | —1.83857 | —1.94457 | —2.05057 |
| $u_0$ | —12.5 | —15.9 | —20.4 | —26.0 | —33.2 | —42.3 | —54.0 | —69.0 | —88.0 | —112.3 |
| $\log w_0$ | 7.13144 | 7.34344 | 7.55544 | 7.76744 | 7.97944 | 8.19144 | 8.40344 | 8.61544 | 8.82744 | 9.03944 |
| $w_0$ | .00135 | .00221 | .00359 | .00585 | .00954 | .01554 | .02532 | .04125 | .06721 | .10951 |

TABLE 72.—*The imperfect dumb-bell-shaped vortex,* $\psi_1$. *Results from the truncating plane* $az = 50°$.

The radii $\varpi$ in kilometers.

| Altitude. | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
|---|---|---|---|---|---|---|---|---|
| ° $az = 180$ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ |
| 170 | 3079.9 | 2412.9 | 1890.3 | 1480.6 | 1160.2 | 909.0 | 712.1 | 557.9 |
| 160 | 2194.6 | 1719.3 | 1346.9 | 1055.2 | 826.7 | 647.7 | 507.4 | 397.5 |
| 150 | 1815.0 | 1422.0 | 1114.0 | 872.8 | 683.7 | 535.7 | 419.6 | 328.8 |
| 140 | 1600.8 | 1254.1 | 982.5 | 769.7 | 603.0 | 472.4 | 370.1 | 290.0 |
| 130 | 1466.4 | 1148.8 | 900.0 | 705.1 | 552.4 | 432.8 | 339.0 | 265.6 |
| 120 | 1379.2 | 1080.5 | 846.5 | 663.2 | 519.5 | 407.0 | 318.9 | 249.8 |
| 110 | 1324.0 | 1037.2 | 821.6 | 636.6 | 498.8 | 390.7 | 306.1 | 239.8 |
| 100 | 1293.3 | 1013.2 | 793.8 | 621.9 | 487.2 | 381.7 | 299.0 | 234.3 |
| 90 | 1283.4 | 1005.5 | 787.7 | 617.0 | 483.5 | 378.8 | 296.7 | 232.5 |
| 80 | 1293.3 | 1013.2 | 793.8 | 621.9 | 487.2 | 381.7 | 299.0 | 234.3 |
| 70 | 1324.0 | 1037.2 | 812.6 | 636.6 | 498.8 | 490.7 | 306.1 | 239.8 |
| 60 | 1379.2 | 1080.5 | 793.8 | 663.2 | 519.5 | 407.0 | 318.9 | 249.8 |
| 50 | 1466.4 | 1148.8 | 787.7 | 705.1 | 552.4 | 432.8 | 339.0 | 265.6 |

The radial velocity $u_1$ in meters per second.

| Altitude. | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
|---|---|---|---|---|---|---|---|---|
| ° $az = 180$ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ |
| 170 | 40.2 | 45.4 | 51.3 | 58.0 | 65.5 | 74.0 | 83.6 | 94.4 |
| 160 | 27.3 | 30.9 | 34.9 | 39.4 | 44.5 | 50.3 | 56.8 | 64.2 |
| 150 | 20.8 | 23.5 | 26.6 | 30.0 | 33.9 | 38.3 | 43.0 | 48.9 |
| 140 | 16.2 | 18.4 | 20.7 | 23.4 | 26.5 | 29.9 | 33.8 | 38.1 |
| 130 | 12.5 | 14.1 | 16.0 | 18.0 | 20.4 | 23.0 | 26.0 | 29.4 |
| 120 | 9.1 | 10.3 | 11.7 | 13.2 | 14.9 | 16.8 | 19.0 | 21.5 |
| 110 | 6.0 | 6.8 | 7.7 | 8.7 | 9.8 | 11.0 | 12.5 | 14.1 |
| 100 | 3.0 | 3.4 | 3.8 | 4.3 | 4.8 | 5.5 | 6.2 | 7.0 |
| 90 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 80 | — 3.0 | — 3.4 | — 3.8 | — 4.3 | — 4.8 | — 5.5 | — 6.2 | — 7.0 |
| 70 | — 6.0 | — 6.8 | — 7.7 | — 8.7 | — 9.8 | —11.0 | —12.5 | —14.1 |
| 60 | — 9.1 | —10.3 | —11.7 | —13.2 | —14.9 | —16.8 | —19.0 | —21.5 |
| 50 | —12.5 | —14.1 | —16.0 | —18.0 | —20.4 | —23.0 | —26.0 | —29.4 |

TABLE 72.—*The imperfect dumb-bell-shaped vortex, $\psi_1$. Results from the truncating plane az=50°*—Continued.

The tangential velocity $v_1$ in meters per second.

| Altitude. | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
|---|---|---|---|---|---|---|---|---|
| ° | | | | | | | | |
| az=180 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 170 | 7.1 | 8.0 | 9.0 | 10.2 | 11.6 | 13.0 | 14.7 | 16.7 |
| 160 | 9.9 | 11.2 | 12.7 | 14.3 | 16.2 | 18.3 | 20.7 | 23.4 |
| 150 | 12.0 | 13.6 | 15.4 | 17.3 | 19.6 | 22.1 | 25.0 | 28.3 |
| 140 | 13.6 | 15.4 | 17.4 | 19.7 | 22.2 | 25.1 | 28.4 | 32.0 |
| 130 | 14.9 | 16.8 | 19.0 | 21.5 | 24.3 | 27.4 | 31.0 | 35.0 |
| 120 | 15.8 | 17.9 | 20.2 | 22.8 | 25.8 | 29.1 | 32.9 | 37.2 |
| 110 | 16.5 | 18.6 | 21.0 | 23.8 | 26.9 | 30.3 | 34.3 | 38.7 |
| 100 | 16.9 | 19.1 | 21.5 | 24.3 | 27.5 | 31.1 | 35.1 | 39.7 |
| 90 | 17.0 | 19.2 | 21.7 | 24.5 | 27.7 | 31.3 | 35.4 | 40.0 |
| 80 | 16.9 | 19.1 | 21.5 | 24.3 | 27.5 | 31.1 | 35.1 | 39.7 |
| 70 | 16.5 | 18.6 | 21.0 | 23.8 | 26.9 | 30.3 | 34.3 | 38.7 |
| 60 | 15.8 | 17.9 | 20.2 | 22.8 | 25.8 | 29.1 | 32.9 | 37.2 |
| 50 | 14.9 | 16.8 | 19.0 | 21.5 | 24.3 | 27.4 | 31.0 | 35.0 |

The vertical velocity $w_1$ in meters per second.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| az=180 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 |
| 170 | .0003 | .0004 | .0006 | .0009 | .0013 | .0019 | .0028 | .0040 |
| 160 | .0006 | .0009 | .0013 | .0018 | .0026 | .0038 | .0054 | .0078 |
| 150 | .0009 | .0018 | .0018 | .0026 | .0038 | .0055 | .0079 | .0115 |
| 140 | .0012 | .0016 | .0024 | .0034 | .0049 | .0071 | .0102 | .0147 |
| 130 | .0014 | .0020 | .0028 | .0041 | .0059 | .0084 | .0122 | .0176 |
| 120 | .0015 | .0022 | .0032 | .0046 | .0066 | .0095 | .0138 | .0198 |
| 110 | .0016 | .0024 | .0035 | .0050 | .0072 | .0104 | .0149 | .0215 |
| 100 | .0017 | .0025 | .0036 | .0052 | .0075 | .0109 | .0157 | .0226 |
| 90 | .0018 | .0025 | .0037 | .0053 | .0076 | .0110 | .0159 | .0229 |
| 80 | .0017 | .0025 | .0036 | .0052 | .0075 | .0109 | .0157 | .0226 |
| 70 | .0016 | .0024 | .0035 | .0050 | .0072 | .0104 | .0149 | .0215 |
| 60 | .0015 | .0022 | .0032 | .0046 | .0066 | .0095 | .0138 | .0198 |
| 50 | .0014 | .0020 | .0028 | .0041 | .0059 | .0084 | .0122 | .0176 |

TABLE 73.—*The perfect dumb-bell-shaped vortex, $\psi_0$.*

The radii $\varpi$ remain the same (see table 72).

The radial velocity $u_0$ in meters per second.

| Altitude. | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
|---|---|---|---|---|---|---|---|---|
| ° | | | | | | | | |
| az=180 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ |
| 170 | 40.2 | 51.3 | 65.5 | 83.6 | 106.7 | 136.2 | 173.8 | 221.9 |
| 160 | 27.3 | 34.9 | 44.5 | 56.8 | 72.5 | 92.6 | 118.2 | 150.9 |
| 150 | 20.8 | 26.6 | 33.9 | 43.3 | 55.3 | 70.6 | 90.1 | 115.0 |
| 140 | 16.2 | 20.7 | 26.5 | 33.8 | 43.1 | 55.1 | 70.3 | 89.7 |
| 130 | 12.5 | 15.9 | 20.4 | 26.0 | 33.2 | 42.3 | 54.0 | 69.0 |
| 120 | 9 1 | 11.7 | 14.9 | 19.0 | 24.3 | 31.0 | 39.5 | 50.4 |
| 110 | 6.0 | 7.7 | 9.7 | 12.5 | 15.9 | 20.3 | 26.0 | 33.1 |
| 100 | 3.0 | 3.8 | 4.8 | 6.2 | 7.9 | 10.1 | 12.9 | 16.4 |
| 90 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 80 | − 3.0 | − 3.8 | − 4.8 | − 6.2 | − 7.9 | −10.1 | −12.9 | −16.4 |
| 70 | − 6.0 | − 7.7 | − 9.7 | −12.5 | −15.9 | −20.3 | −26.0 | −33.1 |
| 60 | − 9.1 | −11.7 | −14.9 | −19.0 | −24.3 | −31.0 | −39.5 | −50.4 |
| 50 | −12.5 | −15.9 | −20.4 | −26.0 | −33.2 | −42.3 | −54.0 | −69.0 |

The tangential velocity $v_0$ in meters per second.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| az=180 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 170 | 7.1 | 9.0 | 11.6 | 14.7 | 18.8 | 24.0 | 30.7 | 39.1 |
| 160 | 9.9 | 12.7 | 16.2 | 20.7 | 26.4 | 33.7 | 43.0 | 54.9 |
| 150 | 12.0 | 15.4 | 19.6 | 25.0 | 31.9 | 40.8 | 52.0 | 66.4 |
| 140 | 13.6 | 17.4 | 22.2 | 28.4 | 36.2 | 46.2 | 59.0 | 75.3 |
| 130 | 14.9 | 19.0 | 24.2 | 31.0 | 39.5 | 50.4 | 64.4 | 82.2 |
| 120 | 15.8 | 20.2 | 25.8 | 32.8 | 42.0 | 53.6 | 68.5 | 87.4 |
| 110 | 16.5 | 21.0 | 26.9 | 34.3 | 43.8 | 55.9 | 71.3 | 91.0 |
| 100 | 16.9 | 21.5 | 27.5 | 35.1 | 44.8 | 57.2 | 73.0 | 93.2 |
| 90 | 17.0 | 21.7 | 27.7 | 35.4 | 45.1 | 57.6 | 73.6 | 93.9 |
| 80 | 16.9 | 21.5 | 27.5 | 35.1 | 44.8 | 57.2 | 73.0 | 93.2 |
| 70 | 16.5 | 21.0 | 26.9 | 34.3 | 43.8 | 55.9 | 71.3 | 91.0 |
| 60 | 15.8 | 20.2 | 25.8 | 32.8 | 42.0 | 53.6 | 68.5 | 87.4 |
| 50 | 14.9 | 19.0 | 24.2 | 31.0 | 39.5 | 50.4 | 64.4 | 82.2 |

The vertical velocity $w_0$ in meters per second.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| az=180 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 |
| 170 | .0003 | .0005 | .0008 | .0013 | .0022 | .0035 | .0057 | .0094 |
| 160 | .0006 | .0010 | .0016 | .0026 | .0043 | .0069 | .0113 | .0184 |
| 150 | .0009 | .0014 | .0024 | .0038 | .0062 | .0101 | .0165 | .0269 |
| 140 | .0012 | .0018 | .0030 | .0049 | .0080 | .0130 | .0212 | .0346 |
| 130 | .0014 | .0022 | .0036 | .0058 | .0095 | .0155 | .0253 | .0412 |
| 120 | .0015 | .0025 | .0041 | .0066 | .0108 | .0176 | .0286 | .0466 |
| 110 | .0016 | .0027 | .0044 | .0072 | .0117 | .0191 | .0311 | .0506 |
| 100 | .0017 | .0028 | .0046 | .0075 | .0123 | .0202 | .0326 | .0530 |
| 90 | .0018 | .0029 | .0047 | .0076 | .0124 | .0203 | .0331 | .0538 |
| 80 | .0017 | .0028 | .0046 | .0075 | .0123 | .0202 | .0326 | .0530 |
| 70 | .0016 | .0027 | .0044 | .0072 | .0117 | .0191 | .0311 | .0506 |
| 60 | .0015 | .0025 | .0041 | .0066 | .0108 | .0176 | .0286 | .0466 |
| 50 | .0014 | .0022 | .0036 | .0058 | .0095 | .0155 | .0253 | .0412 |

TABLE 74.—*The component vortex, $\psi_1 - \psi_0 = \psi_2$. The reversing vortex.*

The radii $\varpi$ remain the same (see table 72).

The radial velocity $u_2 = u_1 - u_0$.

| Altitude. | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
|---|---|---|---|---|---|---|---|---|
| ° | | | | | | | | |
| az=180 | — | — | — | — | — | — | — | — |
| 170 | 0.0 | −5.9 | −14.2 | −25.6 | −41.2 | −62.2 | −90.2 | −127.5 |
| 160 | 0.0 | −4.0 | − 9.6 | −17.4 | −28.0 | −42.3 | −61.4 | − 86.7 |
| 150 | 0.0 | −3.1 | − 7.3 | −13.3 | −21.4 | −32.3 | −47.1 | − 66.1 |
| 140 | 0.0 | −2.3 | − 5.8 | −10.4 | −16.6 | −25.2 | −36.5 | − 51.6 |
| 130 | 0.0 | −1.8 | − 4.4 | − 8.0 | −12.8 | −19.3 | −28.0 | − 39.6 |
| 120 | 0.0 | −1.4 | − 3.2 | − 5.8 | − 9.4 | −14.2 | −20.5 | − 28.9 |
| 110 | 0.0 | −0.9 | − 2.0 | − 3.8 | − 6.1 | − 9.3 | −13.5 | − 19.0 |
| 100 | 0.0 | −0.4 | − 1.0 | − 1.9 | − 3.1 | − 4.6 | − 6.7 | − 9.4 |
| 90 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 80 | 0.0 | +0.4 | + 1.0 | + 1.9 | + 3.1 | + 4.6 | + 6.7 | + 9.4 |
| 70 | 0.0 | +0.9 | + 2.0 | + 3.8 | + 6.1 | + 9.3 | +13.5 | + 19.0 |
| 60 | 0.0 | +1.4 | + 3.2 | + 5.8 | + 9.4 | +14.2 | +20.5 | + 28.9 |
| 50 | 0.0 | +1.8 | + 4.4 | + 8.0 | +12.8 | +19.3 | +28.0 | + 39.6 |

The tangential velocity $v_2 = v_1 - v_0$.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| az=180 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 170 | 0.0 | −1.0 | −2.6 | − 4.5 | − 7.2 | −11.0 | −16.0 | −22.4 |
| 160 | 0.0 | −1.5 | −3.5 | − 6.4 | −10.2 | −15.4 | −22.3 | −31.5 |
| 150 | 0.0 | −1.8 | −4.2 | − 7.7 | −12.3 | −18.7 | −27.0 | −38.1 |
| 140 | 0.0 | −2.0 | −4.8 | − 8.7 | −14.0 | −21.1 | −30.6 | −43.3 |
| 130 | 0.0 | −2.2 | −5.2 | − 9.5 | −15.2 | −23.0 | −33.4 | −47.2 |
| 120 | 0.0 | −2.3 | −5.6 | −10.0 | −16.2 | −24.5 | −35.6 | −50.2 |
| 110 | 0.0 | −2.4 | −5.9 | −10.5 | −16.9 | −25.6 | −37.0 | −52.3 |
| 100 | 0.0 | −2.4 | −6.0 | −10.8 | −17.3 | −26.1 | −37.9 | −53.5 |
| 90 | 0.0 | −2.5 | −6.0 | −10.9 | −17.4 | −26.3 | −38.2 | −53.9 |
| 80 | 0.0 | −2.4 | −6.0 | −10.8 | −17.3 | −26.1 | −37.9 | −53.5 |
| 70 | 0.0 | −2.4 | −5.9 | −10.5 | −16.9 | −25.6 | −37.0 | −52.3 |
| 60 | 0.0 | −2.3 | −5.6 | −10.0 | −16.2 | −24.5 | −35.6 | −50.2 |
| 50 | 0.0 | −2.2 | −5.2 | − 9.5 | −15.2 | −23.0 | −33.4 | −47.2 |

The vertical velocity $w_2 = w_1 - w_0$.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| az=180 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 |
| 170 | .0000 | −.0001 | −.0004 | −.0009 | −.0016 | −.0029 | −.0054 | −.0095 |
| 160 | .0000 | −.0001 | −.0008 | −.0017 | −.0031 | −.0059 | −.0106 | −.0187 |
| 150 | .0000 | −.0001 | −.0012 | −.0024 | −.0046 | −.0086 | −.0154 | −.0274 |
| 140 | .0000 | −.0002 | −.0015 | −.0031 | −.0059 | −.0110 | −.0199 | −.0352 |
| 130 | .0000 | −.0002 | −.0017 | −.0036 | −.0071 | −.0131 | −.0236 | −.0419 |
| 120 | .0000 | −.0003 | −.0020 | −.0042 | −.0081 | −.0148 | −.0268 | −.0474 |
| 110 | .0000 | −.0003 | −.0022 | −.0045 | −.0087 | −.0162 | −.0291 | −.0513 |
| 100 | .0000 | −.0003 | −.0023 | −.0048 | −.0093 | −.0169 | −.0301 | −.0539 |
| 90 | .0000 | −.0004 | −.0023 | −.0048 | −.0093 | −.0172 | −.0309 | −.0546 |
| 80 | .0000 | −.0003 | −.0023 | −.0048 | −.0093 | −.0169 | −.0301 | −.0539 |
| 70 | .0000 | −.0003 | −.0022 | −.0045 | −.0087 | −.0162 | −.0291 | −.0513 |
| 60 | .0000 | −.0003 | −.0020 | −.0042 | −.0081 | −.0148 | −.0268 | −.0474 |
| 50 | .0000 | −.0002 | −.0017 | −.0036 | −.0071 | −.0131 | −.0236 | −.0419 |

The total velocity $q_2 = q_1 - q_0$.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| az=180 | — | — | — | — | — | — | — | — |
| 170 | 0.0 | −6.0 | −14.4 | −26.6 | −41.8 | −63.2 | −91.6 | −129.4 |
| 160 | 0.0 | −4.2 | −10.3 | −18.6 | −29.8 | −45.0 | −65.3 | − 92.2 |
| 150 | 0.0 | −3.5 | − 8.5 | −15.3 | −24.7 | −37.2 | −54.0 | − 76.3 |
| 140 | 0.0 | −3.1 | − 7.5 | −13.5 | −21.7 | −32.9 | −47.7 | − 67.3 |
| 130 | 0.0 | −2.8 | − 6.9 | −12.4 | −19.9 | −30.1 | −43.6 | − 61.6 |
| 120 | 0.0 | −2.7 | − 6.5 | −11.5 | −18.7 | −28.3 | −41.0 | − 58.0 |
| 110 | 0.0 | −2.6 | − 6.2 | −11.2 | −18.0 | −27.2 | −39.4 | − 55.7 |
| 100 | 0.0 | −2.5 | − 6.0 | −10.9 | −17.6 | −26.6 | −38.5 | − 54.3 |
| 90 | 0.0 | −2.5 | − 6.0 | −10.9 | −17.5 | −26.3 | −38.2 | − 53.9 |
| 80 | 0.0 | −2.5 | − 6.0 | −10.9 | −17.6 | −26.6 | −38.5 | − 54.3 |
| 70 | 0.0 | −2.6 | − 6.2 | −11.2 | −18.0 | −27.2 | −39.4 | − 55.7 |
| 60 | 0.0 | −2.7 | − 6.5 | −11.5 | −18.7 | −28.3 | −41.0 | − 58.0 |
| 50 | 0.0 | −2.8 | − 6.9 | −12.4 | −19.9 | −30.1 | −43.6 | − 61.6 |

### THE REVERSING VORTEX AND THE COMPOSITION OF VORTICES.

By comparing the perfect vortex $\psi_0$, of Table 73, with the imperfect vortex $\psi_1$, of Table 72, it is seen that a reversing vortex, $\psi_1 - \psi_0 = \psi_2$, added algebraically to $\psi_0$ will produce $\psi_1$. This vortex rotates in the opposite direction to the observed vortex and it descends from the clouds to the surface. The radii $\varpi_n$ of these vortices $\psi_1$, $\psi_2$, $\psi_0$, are assumed to be identical. This reversing vortex is equivalent to a system of forces which are operating to retard the imperfect vortex $\psi_1$ so that it can not reach the perfect vortex $\psi_0$ in its development. These resisting forces are (1) the simple friction; (2) the internal minor rotations and inertia vortices; (3) the asymmetrical distribution of temperatures so that they do not readily produce vortex motion, but nearly horizontal currents which are much less fully organized than in symmetrical vortices. This method of determining the total forces of resistance at any point may have great value in meteorology; it promises to become a fruitful method of discussing the difficult problems of friction and inertia in the atmosphere.

Instead of deriving the resistance vortex by computing the imperfect and the perfect vortices separately, we can proceed directly from the formulas. If the values of $A_2$ in the reversing vortex be computed from the values of $u_2$, $v_2$, $w_2$, produced by composition, it is found to be the same as would be obtained by subtracting the constants, $A_1 - A_0 = A_2$, at once.

| Imperfect vortex, | Perfect vortex, | Reversing vortex, |
|---|---|---|
| $u_1 = -A_1 a\varpi \cos az$, | $u_0 = -A_0 a\varpi \cos az$, | $u_2 = -(A_1 - A_0) a\varpi \cos az$, |
| $v_1 = A_1 a\varpi \sin az$, | $v_0 = A_0 a\varpi \sin az$, | $v_2 = (A_1 - A_0) a\varpi \sin az$, |
| $w_1 = 2A_1 \sin az$. | $w_0 = 2A_0 \sin az$. | $w_2 = 2(A_1 - A_0) \sin az$. |

The values of $(A_1 - A_0)$ found from $u_2$, $v_2$, $w_2$, respectively, are identical. In other words, by subtracting the constants $A_0$ in Table 71 from $A_1$ in Table 70 the values of $(A_1 - A_0)$ will be found which will reproduce the vortex of Table 74, the signs being as there computed. It follows, from this exposition, *that a vortex can be analyzed into its components, or the resultant vortex can be found from its components.* If, on a given level $az$, the velocities $u$, $v$, $w$, are observed in a vortex whose vertical dimensions are determined thru the $a$-constant, we shall find values of $A$ which may or may not be in harmony with each other. If we similarly compute the values of $A$ on different levels for the proper radii $\varpi_n$, their divergence or agreement will permit a discussion of the internal forces that have caused them to change from a given vortex type. Since the land-cyclones are apparently based upon the dumb-bell-shaped vortex, tho it has been much depleted, it yet offers us a method of studying several difficult problems. The Cloud Report contains much data in form for such an application, and it is therefore of interest to consider to what extent the observations, as there discust, are in harmony with this vortex. Since the forces producing the motion in a cyclone are entirely different from those in a hurricane it is not probable that there will be any close agreement. It would leave an erroneous impression to suppose that the ordinary dumb-bell vortex, even if it be defective, is still to be found in the land-cyclone, because the vortex there represented is a concave dumb-bell vortex and not a convex vortex, as already illustrated in the tornado and the hurricane. The ocean-cyclone has been used merely as a transition type between the concave and the convex types of dumb-bell vortices.

## A BRIEF HISTORICAL REVIEW OF THE THEORIES OF STORMS.

Cyclones and anticyclones have usually been treated as masses of air with warm and cold centers, respectively, by meteorologists. Two theories stand in contrast to this general view: (1) Dove's theory of the mechanical interference of currents moving in different directions; and (2) Bigelow's thermodynamic asymmetric currents of different temperatures which produce cyclonic and anticyclonic circulations with their centers near the edges of these currents. As Dove's theory deals with long currents it will be proper to recall the exact status of his theory. Dove wrote with much perspicacity for a great period of time, 1827–1873, wherein he was a collector of facts observed in various parts of the world, which were discust in a descriptive manner rather than by a mathematical method. His "Law of Storms," second edition, 1862, translated by Robert H. Scott, is most easily accessible to English readers, and the following references are made to that book. All meteorology is to be classified under thermodynamics and circulation, and a few extracts will make Dove's views plain on both points.

(1) *Thermodynamics.*—This subject is distributed into the cases of circular masses, and of parallel streams.

(Page 18.) If any point in a liquid be heated more strongly than the others, currents arise in it, and the colder particles flow from all sides toward this heated point.



(Page 256.) If two currents, on coming in contact with each other, have altered their paths thru any angle, so that they flow in opposite directions in parallel channels, the following question arises: What conditions will cause mutual lateral displacement after such a state of things as that described is once in existence? The most obvious cause is to be found in the fact that the cold air of the polar current exerts a greater lateral pressure than the warm air of the equatorial current and therefore has a tendency to displace it.

(Page 79.) The equatorial current flows from a warm to a cold, and the polar current from a cold to a warm region. The characteristic differences of these currents may always be traced to their differences in temperature, and to differences in the action exerted by the earth (gravitation) on them in their course.

(Page 83.) As regards the mutual displacement of the currents, I have ascertained, from observations which have been carried on at Königsberg for a long series of years, that the southerly current displaces the northerly in the upper strata of the atmosphere before it does so in the lower strata; while the displacement of the southerly by the northerly takes place first in the lower and afterwards in the upper.

These displacement propositions are, of course, the commonplaces of the thermodynamics of fluids of different temperatures. Unfortunately meteorologists have followed up the central-mass system, to the exclusion of the parallel-current system, in the analytical discussions, and the results have been disappointing. The second equation of motion readily yields central solutions, while the hydrodynamics of the parallel-current systems is exceedingly difficult to work out in a practical form. It is easy to state the general thermodynamic problem, but hard to make applications in the atmosphere. Margules has given several cases for model conditions, particularly under adiabatic transformations, but in the free air adiabatic conditions prevail over such limited spaces before breaking up into minor mixing vortices, that they are not readily transferred to the problems of cyclones and anticyclones. Bigelow has sought to point out some modifications in the thermodynamic formulas, and especially to collect the observations in the several strata up to 10,000 meters, in such a form as to make a beginning in the study of the real thermodynamics of the atmospheric circulation.

(2) *Circulation.*—In his treatment of the circulation of air over the hemisphere and in local cyclones, Dove has not been so happy as in his thermodynamic suggestions. He discusses (1) radial motions toward a center below, and away from a center above; (2) tangential motions around an axis; (3) motions in straight parallel lines; (4) motions to and from the apex of the sector formed by the converging meridians. He is chiefly interested in establishing a "law of gyration," by which the circulation is directed from the south thru west, north, and east. The chief if not the sole application of his thermodynamics of lateral expansion, is in the justification of this rule, but he says:

(Page 175.) When I first published my papers on the winds, I was disposed to refer the law of gyration, as well as the rotary motion of storms, to the mutual interference of two currents of air, which alternately displace each other in a lateral direction A closer examination of the phenomena showed me that the law of gyration depended on more general principles, and that it was a simple and necessary consequence of the motion of the earth on its axis.

He thereby stated the practical effects of the "deflecting force" due to the earth's rotation, but abandoned the lateral displacement theory as regards the formation of storms. Dove's mechanical theory of interference of currents flowing in different directions is stated in many places, and seems to have been derived from his conception of the action of the upper and lower trade-winds in producing hurricanes which form the initial stage in cyclones of the higher latitudes, and are analogous in their formation under all circumstances.

(Page 176.) It was not until later that I was able to supply this deficiency by proving that a cyclonic movement was produced whenever the interposition of any obstacle interfered with the regular change in the direction of the wind, which is due to the rotation of the earth.

(Page 185.) This renders it probable that the primary cause of the West India hurricanes is the intrusion of a portion of the upper trade-wind into that which lies underneath it.

(Page 188.) The interference of a current flowing from east to west with another which is flowing from southwest to northeast must necessarily generate a rotatory motion in the direction opposite to that of the hands of a watch. According to this view the hurricane which advances from southeast to northwest in the under trade-wind represents the advancing point of contact of two currents in the upper strata which are moving in directions at right angles to each other. This is the primary cause of the rotatory motion.

(Page 196.) The upper part of the cyclone will accordingly dilate at once and advance in a direction different from that of the other part. Hence, as a secondary phenomenon, a suction will ensue in the center of the cyclone and, also, a diminution of pressure over the surface of the earth.

In regard to the general circulation, Dove conceives the vertical section as a figure $\infty$, with the ascending currents *both at the equator and poles*, crossing in the outer limit of the Tropics.

(Page 271.) The upper counter-trade of the Torrid Zone descends at the outer edge of this area; flows into the Temperate Zone; rises again when it comes into higher latitudes; flows back as a polar current in the upper strata of the atmosphere of the Temperate Zone; descends afresh at the Tropics; flows in toward the equator along the surface of the ground as the ordinary direct trade-wind, and at the equator rises again.

(Page 221.) The West India hurricanes are due to the interference of lateral cross-currents with the upper trade-wind on its return from the equator, portions of which, being forced to enter the lower strata of the atmosphere, meet with a constant wind, moving in a direction opposite to their own and thus produce a cyclone. Outside the trade-wind area the upper current descends to the surface of the earth, and is predominant there in different districts at different times, while the under-current in the opposite direction is not constant. Here, then, we shall find that the conditions of interference will be constantly presented, but the currents will be directly opposed to each other so that they will only check each other's progress.

It is evident that Dove's theory of interference and obstruction for the formation of cyclones induced him to describe a circulation over the hemisphere which is partially correct in the temperate zones, but erroneous in the polar zones. Since the ordinary canal theory of the circulation with a poleward current from the equator to the pole in the upper strata, and a return current from the pole to the equator in the lower levels (Ferrel, Oberbeck), could produce no currents of different temperatures on the same levels, whether high or low, therefore in the Cloud Report I described the "leakage current," escaping from the Tropics in certain longitudes, as on the western side of the Atlantic high area, by which warm currents are thrown into the United States from the south to meet the cold currents from the north in the lower strata. The theory of interference and obstruction was rejected, and a theory of the asymmetric cyclone and anticyclone described, depending on the lateral interpenetration of the warm and cold masses, thus using the key which Dove threw away, and which is evidently the only key to unlock this problem. Compare Cloud Report, Charts 20–35, p. 606–609, 612, 615–633, and numerous papers in the MONTHLY WEATHER REVIEW.

Ferrel sought the explanation for cyclones in a cold- or warm-centered vortex, as did Guldberg and Mohn, Oberbeck, and others, using different solutions of the second equation of motion, but the source of heat energy at the center is so inconsistent with the observed distribution of the temperature, that the symmetrical cyclone was abandoned by me in favor of the asymmetrical cyclone and anticyclone. The difficulty in making progress with this view was due to the fact that for several years following 1898 reliable temperature observations in the *free* upper strata were not available as they have since become thru the reports of balloon and kite ascensions. Those that have been obtained show that the asymmetric temperature distribution found at the surface presists to the top of the cyclonic disturbance of the eastward drift in substantially the same relations, so that the problem can now be resumed. The preceding papers of this series, as well as the "Studies on the Thermodynamics of the Atmosphere," 1907, are simply introductory to the cyclonic problem, which will no doubt require much careful mathematical study.

It is easy to state a theory in general terms of the action of warm and cold masses on each other in horizontal directions, but to pass thru the thermodynamic equations to the dynamic stream-lines in so complex a system of flow, involving hydrodynamic conditions which are neither simple nor constant, is a work of extreme difficulty. Hann found evidences that the temperatures in the levels above the surface layers did not conform to the central or symmetrical theory of Ferrel, but he believed that the change of temperature, from warm to cold in cyclones, for example, was a dynamic effect such as is produced in ascending currents. The asymmetric theory calls for no such dynamic action to produce thermal effects, but, on the contrary, it takes the observed existing thermal conditions, and finds from them the necessary cyclonic dynamic circulations. In the preceding papers of this series we have been able to trace certain tornadoes and the hurricane to the simple dumb-bell vortex, but in the ocean-cyclone there is evidence that the vortex is becoming very imperfect, so that the dumb-bell vortex must be greatly modified to be applicable to the ocean-cyclone, and so much the more, to the land-cyclone. One of the chief labors of the Cloud Report, 1896–97, was to compute the radial component of the velocity, $u_2$, from the cloud observations, and this result, Table 126, p. 626 of that report, will be employed in the following paper on the land cyclones to bring out some of the leading features of this difficult problem.

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## DEFICIENT HUMIDITY INDOORS.

By F. H. DAY, B. Sc., Demonstrator of Physics, McGill University.
Dated Montreal, December 21, 1908.

In view of the republication[1] of Prof. R. DeC. Ward's observations on the relative indoor and outdoor hygrometric conditions in the neighborhood of Boston and Cambridge, Mass., it is interesting to compare these with some similar investigations carried on in Montreal during the winters of 1906 and 1908 by Dr. T. A. Starkey and Dr. H. T. Barnes,[2] of McGill University. This work was undertaken in order to determine the effect of a successively dry atmosphere on the human organism—a question of great moment in Canada, especially in those parts where during the winter months the heating of the houses necessitates the heating of the indoor atmosphere, thereby causing a tremendous drying of the air. And this is the first condition for the difference between the air in the two places. Professor Ward was working with an indoor and outdoor difference of about 35° F., while in Montreal this increases to a difference of from 70° to 100° F. during the colder months. Knowing that if we warm a given quantity of air completely saturated with water vapor at the initial temperature, it no longer remains saturated, we can readily see what an enormous change in the relative humidity is entailed by heating outside air at a temperature of 0° F., or below, to a higher temperature.

As a danger to the health caused by this excessive dryness, Professor Ward mentions the strain put upon the body going out from a high temperature and very dry atmosphere into an atmosphere low in temperature and comparatively high in humidity. Starkey and Barnes find another and quite as serious a danger in the direct effect of the dry atmosphere itself. This condition is well borne out by some striking illustrations. To quote from this paper:[2]

The action of a dry atmosphere affects primarily the mucous membranes lining the respiratory tract, chiefly that of the nose, the throat, and the bronchial tubes. It is a peculiarly mechanical irritant, resulting in a condition of congestion of the mucous membranes before mentioned. If this irritation be continued for any length of time these swollen membranes with difficulty regain their normal state. We have thus all the conditions favorable for a chronic catarrh, and this chronic condition being established we get all the typical symptoms of nasal-pharyngeal

[1] Monthly Weather Review, September, 1908, 36: 281.
[2] Trans. Roy. Soc. Can.